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MATHEMATICAL MODELLING AND FINITE ELEMENT ANALYSIS OF SUSPENSION FORK FOR TWO WHEELERS

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ABSTRACT

The steering column is constructed by front suspension forks in most two wheelers. The paper focuses on the Finite Element Analysis of the suspension fork for motor bike, and developing an empirical equation in terms of internal diameter and central bush wall thickness. Finite element analysis has been done to verify the analytical analysis done with the help of thick wall theory. An empirical relation is necessary as different combinations of internal diameter and bush wall thickness are available in the industry. The empirical relation can be utilized to design a new fork for two wheelers.

KEYWORDS: Suspension, Fork, Hoop Stress, Correction Factor

INTRODUCTION

Suspension is used in automobile for driving safety and increase the ride quality. Overall, it consists of tires, spring, shock absorber, springs, dampers and linkage connecting vehicles and wheels. It mostly uses the strain energy of spring to absorb the vibration during riding. In two wheeler, suspension system consist of a fork which is having a camped joint between the wheel clamp and wheel pin at one end and a bolted joint between the a crown and outer tube of the fork. An interference fit exist between steering shaft and crown. The designing and study of this inference joint is very important as the whole reliability and safety of suspension system is dependent on this joint. If the interference and friction is not enough between the crown and steering shaft, the steering shaft may slide causing a serious problem. This fork has an inner tube and outer tube which slides relative to each other. This tube consists of a back spring (for adjusting the overall stiffness of the suspension system), a front spring and a damping fluid. When the tubes slides relative to each other the spring get compress absorbing the vibration energy, hence only an limited amount of transmitted force is allowed to get transmitted to the driver and damping fluid damps the vibration.

LITERATURE REVIEW

The insufficient information regarding the friction coefficient and the average hub pressure hinders the designing of the steering shaft and fork coupling. The factors mentioned above which are unknown and the clamping surface which is known influences the basic design parameter called the dehub pressure, which equals the product of friction coefficient, hub pressure and the surface area. Of late, In consideration of the safety of the rider, with the massive increase in the weight and the power of the two wheelers, there is an increase in the dehub pressure. The major critical design parameters such as the amount of interference and its proportional factor hub pressure must be leveled at higher values which in turn should not overcome the yielding of the components, but the factor should exceed the releasing tests. Hence a generalized method has been developed to calculate the friction coefficient and the hub pressure with more accuracy. The assumption of the friction coefficient is done by using the Design for Experiment methodology. This is carried out to exaggerate the information about the experimental data. Herewith two material combinations have been considered such as steel-aluminum and aluminum-aluminum.

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The other issue is that the component stiffness changes with the radial coordinate of the bush. This occurs because the coupler isn't axially symmetric. Hence fem analysis is necessary to define the tensile stress on the coupling surface area.

Fig 1: showing the fork coupling & the steering shaft

DESIGN

Our reviews were devoted to figure out a general scientific law, capacity of some geometric parameters, imparted by every sort of fork, and ready to right the hypothetical equations. The FEM examination is vital on the grounds that the arrangement gave by the coinciding and balance comparisons is not successful when connected to asymmetrical components, for example, the fork. The tension on the coupling range ends up being neither steady nor calculable as normal value if the Thick Walled Cylinders hypothesis (Lamè's mathematical statements) is implemented. The fem analysis also proves that the radial and the tangential stresses prove to be varying around the bush in the fork coupling. Henceforth it has become necessary to perform fem analysis over dimensionally varying fork coupling designs. This is done to calculate the hub pressure and the tangential stress and to impart the necessary design parameters in the theoretical formulae.

Hub pressure,

$$P = \alpha / \{ \frac{1}{E_H} (\frac{1 + Q_H^2}{1 - Q_H^2} + \upsilon_H) + \frac{1}{E_s} (\frac{1 + Q_S^2}{1 - Q_S^2} - \upsilon_s) \}$$

$$\text{Radial stress, } \sigma_r = P$$

$$\text{Tangential stress, } \sigma_t = P(\frac{1+Q^2}{1-Q^2})$$

$$\sigma_r < 0;\ \sigma_t > 0;\ |\sigma_r| < \sigma_t$$

$$\alpha = \frac{actual\ interference\ (z)}{Coupling\ diameter\ (D_H)}$$

$$Q_S = \frac{internal\ diameter\ of\ the\ steering\ shaft}{external\ diameter\ of\ the\ steering\ shaft}$$

$$Q_H = \frac{internal\ diameter\ of\ the\ fork\ hub}{external\ diameter\ of\ the\ fork\ hub}$$

$$U = (D_{Se}) - (D_{Hi})$$

$$Z = (U) - (G)$$

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υ and E are the young's modulus and the poisons ratio of the fork-hub and the steering shaft respectively represented with subscripts H & S. Q_S and Q_H are the ratio of the internal diameter and the external diameter of the steering-shaft and the fork-hub respectively. U is the nominal interference assumed as 1mm for calculation purpose. The actual interference can be calculated using the roughness parameters of the steering shaft and the fork hub. In this study roughness values are assumed to as they are constants in context of calculating stress in the part under study. Roughness value for pipe is assumed to be 1.08 while for hub it is 2.18 on which G value is depend. It is taken as 0.8 times of summation of both roughness values of pipe and hub.

A detailed study has been conducted on the various designs in the coupling. Combinations of three different internal diameter of the coupling 'D' to the five different values of the wall thickness's' are designed and then analyzed for hoop stress and hub pressure.

$$D = \{25, 29, 33\ mm\}$$

$$s = \{6, 7.3, 8, 8.5, 9\ mm\}$$

After detailed fem analyses a correction factor has been determined for the overall pressure developed on the coupling to resolve the issues faced by the asymmetrical models. Upon observing the theoretical values and the fem values it's viable that there are errors in the formulae. Henceforth a beta correctional factor have been introduced into the theoretical formulae for hub pressure and the longitudinal & transverse stresses developed on the bush wall which will thereby give a similar result.

$$P = \left(\frac{Z}{D} . E_H\right) / \left(\frac{1+Q_H^2}{1-Q_H^2} + \upsilon_H\right);$$

$$\sigma_t = (P) . \left(\frac{1+Q^2}{1-Q^2}\right);$$

$$Q_H = \frac{D}{D + 2.s}$$

Now the hub pressure and the stresses from the fem analysis were evaluated and it's been deduced that the values of the theoretical and fem stresses are varying in accordance with the Z value. In order to meet equilibrium between the theoretical and the fem stresses a beta correctional factor which is influenced by the internal diameter D and the bush wall thickness s.

$$^{\beta}{}_{r\_fem} = (\sigma_{r\_fem}) . (\sigma_r)^{-1} = (P_{fem}) . (P)^{-1};$$

$$^{\beta}{}_{t} = (\sigma_{t\_fem}) . (\sigma_t)^{-1}$$

Fig 2: sample fork coupling of D=33mm, s=9mm

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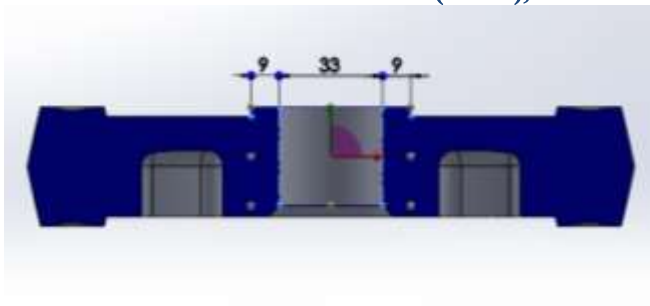

Fig 3: Basic parameters for the computation of the beta value say, D=33mm and s=9mm

Thus in order to avoid the errors created by using the theoretical formulas because of the fork coupling being asymmetrical. The previously mentioned theoretical formula have been corrected and reframed with the deduced beta factor which will now correct the coinciding and balance comparisons in the equations.

METHODOLOGY

FEM analysis:

For doing numerical analysis on the crown, workbench interference of ANSYS v.15 has been used. Solidwork file format of the fifteen crown model has been converted into parasolid neutral format (x_t). Since parasolid is one of the best neutral format with minimum import error and our model contain lots of rendering and fillet. For doing the meshing of the crown, tetrahedral mesh has been used. The reason for doing tetrahedral mesh is:

1. It is volume mesh element. It is used to mesh solid and SOLID45 given as the element of our model.
2. Solids can also be mesh by hexahedral, tetrahedral and pentahedral. In fact, hexahedral mesh will less computational time to solve. But hexahedral does not mesh or comply with geometry with the complex model. In other words, hexahedral not flexible enough to mesh complex geometry. Tetrahedral mesh requires more computational time than hexahedral mesh but it is more flexible to mesh complex model. Pentahedral mesh requires more computational time but is the more flexible also. Tetrahedral is best mesh for the crown models since the crown models contain holes, fillets and other complex entities. Pentahedral mesh can also be used to mesh the crown models, but it will require more computational time and flexibility of tetrahedron mesh is enough to mesh the models.

A tetrahedral mesh contain ten nodes as shown in figure and each node has three DOF in x, y and z direction, thus it contain a total thirty DOF in space.

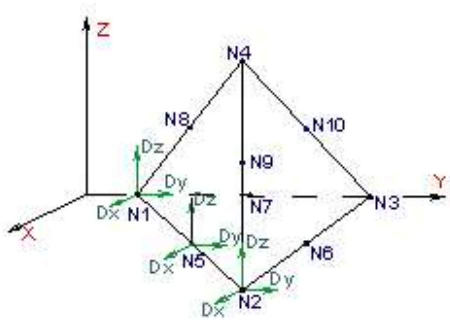

Fig 4: Tetrahedral Mesh Element

A fine tetrahedral mesh of size 3mm on the volume near hub area by defining a sphere of influence of radius 38mm and a coarse mesh of 5 mm on the remaining volume has been done. This has to be done because the volume near the hub is the main area of interest where radial and hoop stress has to be found and the remaining areas are least important. By meshing in the above manner, computational time has been minimized. The meshed model of crown is shown in the figure.

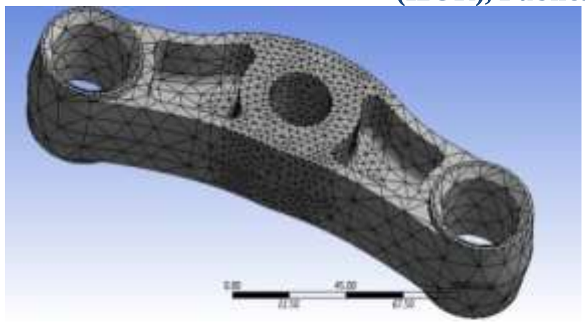

Fig 5: Mesh Model

For getting hoop and radial stress, cylindrical analysis has to be done. For doing this, a cylindrical co-ordinate at the center of the hub at the mid plane of the crown with principal axis as x axis and the y as the secondary axis along the tangential direction as shown in figure has to be defined.

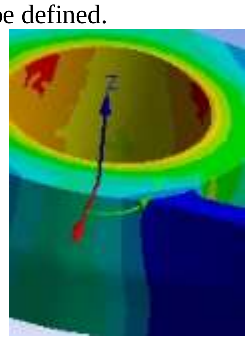

Fig 6: Cylindrical co-ordinate centre

This cylindrical co-ordinate will also act as the center of the sphere of influence while creating mesh.
Now by analyzing the working and construction of the front suspension, an interpretation can be done that there is no displacement between the two holes at the side of crown and the fork. Also, there will be no deformation of the two holes at the sides of the crown. Thus, the interior cylindrical surface of the holes is fixed and this will use as a boundary condition.
While calculating analytical P_f analytically, an interference fit has been considered and material of both crown and steering shaft. So, this P_f is applied to the external surface of steering shaft and also, to the interior surface of the hub. So, if only apply P_f on the interior surface of the hub, it will act as same way as the interference fit. So, only the parasolid file format of the crown without the shaft has been imported and P_f is applied on the interior surface of hub. That will be the second boundary condition. Note that, interior surface of hub is not fixed as in interference fit, the nominal diameter of hub tends to increase that mean there will deformation. The pressure P_f will be in radial direction normal to the interior surface of hub.
While calculating the hoop stress in the crown, the cylindrical co-ordinate system that defined earlier has been used. If the evaluation of solve normal stress with orientation in Y-direction of the cylindrical co-ordinate is done, the hoop stress will be the result as Y axis is tangential in direction. In the same manner, if evaluation of normal stress is done with orientation in X-direction of the cylindrical co-ordinate, the radial stress will be the result. This radial stress will be negative as radial stress is always opposite to applied internal pressure which is in direction of positive X-axis of cylindrical co-ordinate. The figure shows the approximate pattern of hoop and radial stress respectively.

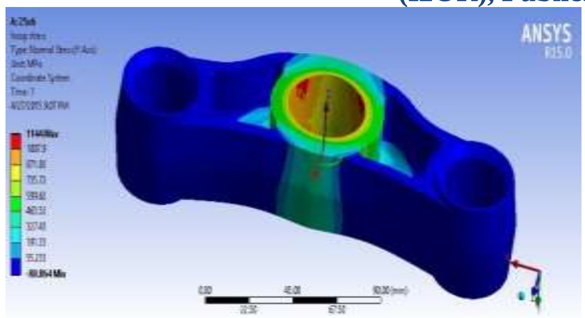

Fig 7: Hoop stress

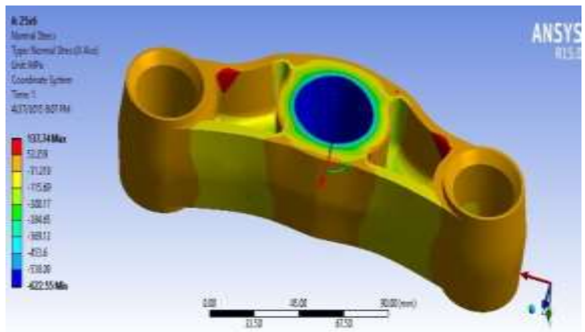

Fig 8: Radial stress

COMPARISON OF ANALYTICAL AND FEM RESULTS

Dimension	σ_{r_FEM} (MPa)	σ_{t_FEM} (MPa)	σ_r(MPa)	σ_t(MPa)	Error in σ_r	Error in σ_t
25x6	-93.7556	172.2857	-91.7451	245.886707	2.0105571	73.6010101
25x7.3	-107.8397	194.0172	-99.4524	231.259827	8.3873054	37.2426383
25x8	-112.517	198.8062	-102.903	224.710937	9.6141319	25.904688
25x8.5	-114.1468	201.2309	-105.125	220.494606	9.0219166	19.2637075
25x9	-115.743	203.8965	-107.169	216.614897	8.5739443	12.718389
29x6	-76.9788	164.5148	-70.9194	212.927115	6.0594563	48.4123458
29x7.3	-83.6654	173.0538	-77.203	199.712835	6.4624601	26.6590807
29x8	-86.921	178.0687	-80.0403	193.746157	6.8811648	15.6774435
29x8.5	-88.815	180.6741	-81.8755	189.886836	6.9405341	9.2127533
29x9	-92.1246	183.5204	-83.5703	186.322642	8.5543694	2.80223101
33x6	-62.1086	153.0692	-56.3211	187.376024	5.7875756	34.3068084
33x7.3	-68.5347	161.3371	-61.5041	175.338587	7.030686	14.0014651
33x8	-72.0798	167.7075	-63.8591	169.869078	8.2207862	2.16160198
33x8.5	-71.0106	165.0569	-65.3876	166.319057	5.6229964	1.26213042
33x9	-72.7470	169.6803	-66.8032	163.0315	5.9438821	6.648828

Table 1: The following table shows the results of analytical and FEM study

From the table it can be easily seen that the trend follow by the stress calculated from analytical formula and stress calculated by numerical analysis follow a similar trend. It can also observed that the analytical stress that are calculated using standard formulae does not match with the FEM stress resulted from ANSYS and the error is about 30-40% in hoop stress and 10-11% in radial stress. To match the solution, correction factor (β) can be defined such that:

$\beta_r = \sigma_{r\_fem}/\sigma_r$
$\beta_t = \sigma_{t\_fem}/\sigma_t$
β value for different crown models evaluated as shown in the table:

Dimension	B_r	β_t
25x6	1.02191	0.7006
25x7.3	1.08433	0.8389
25x8	1.09349	0.88472
25x8.5	1.08582	0.91263
25x9	1.08000	0.94128
29x6	1.08544	0.77263
29x7.3	1.08370	0.86651
29x8	1.08597	0.919083
29x8.5	1.08476	0.951438
29x9	1.10236	0.98496
33x6	1.10276	0.8169
33x7.3	1.11431	0.920146
33x8	1.12849	0.97167
33x8.5	1.08622	1.00834
33x9	1.08897	1.0407

Table 2: β Value

Now, an interpolation of the β value is done into an approximate linear equation in term of diameter and rim size of the hub, so that it can be use it in a generalize way to correct the analytical formula to get a generalize formula. The term approximate is used because stress value is slightly scattered and will get an approximate plane. With the help of Mathematical calculation, following generalized β equations are derived.
For Radial stress:
β_r=0.695094+0.0063664Di_f+0.0279433sFor
Tangential Stress:
β_t=-0.1762+0.0170Di_f+0.0751s

Now, the β equation has to be checked to see whether it gives a satisfactory result or not. For doing this, the analytical formula is interpolated in term of β and the following formulae are derived:
$\sigma_{t\_actual} = \beta_t * \sigma_t$
$\sigma_{r\_actual} = \beta_r * \sigma_r$
The table 3 below shows the results of stress that has been calculated using the above formulae. When it is compared with the stress obtain by numerical method, it can be easily seen that both the stress are more and less similar (it will not be same since, approximate functions are used) and error is about 8% only.

Dimension	$\beta_{r\_Formula}$	$\beta_{t\_formula}$	σ_{r_actual}	σ_{t_actual}	Error in σ_r after correction	Error in σ_t after correction
25x6	1.02191	0.6994	-93.7556	171.9732	7.84635E-05	0.181405
25x7.3	1.08433	0.79703	-105.245	184.321	2.406524517	4.997582
25x8	1.09349	0.8496	-110.909	190.9144	1.429311297	3.969612
25x8.5	1.08582	0.88715	-114.772	195.6118	0.54807482	2.792369
25x9	1.08000	0.9247	-118.502	200.3038	2.38331569	1.762028
29x6	1.08544	0.7674	-74.2795	163.4003	3.506594147	0.677447
29x7.3	1.08370	0.86503	-83.6654	172.7576	0.000152165	0.171138
29x8	1.08597	0.9176	-88.3057	177.7815	1.59255866	0.161308
29x8.5	1.08476	0.95515	-91.4743	181.3704	2.99310038	0.38541
29x9	1.10236	0.9927	-94.5355	184.9625	2.61692097	0.78578
33x6	1.10276	0.8354	-60.4238	156.5339	2.71276651	2.2635
33x7.3	1.11431	0.93303	-68.2186	163.5962	0.461380378	1.4002
33x8	1.12849	0.9856	-72.0798	167.423	0.000143145	0.169648
33x8.5	1.08622	1.02315	-74.7187	170.1693	5.22179675	3.09737
33x9	1.08897	1.0607	-77.2695	172.9275	6.21674141	1.91371

Table 3: Final Result

CONCLUSION

This paper deal with formulating a formula to design an asymmetric crown as axisymmetric thick plate formulae cannot be applicable to crown model, Formula is formulated with help of study of analytical formula and then comparing it with the FEM result has done, verified the result by comparing with the numerical results of the crown model of varying dimensions with different diameters and rim size. By comparing numerical result with the analytical result correction factor calculated and interpolated this correction factor into linear equation in term of diameter and rim thickness of hub.

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